Work Interactions

A work interaction between two systems occurs when their boundary moves under the action of a force.

$$đw = F\,dx \Rightarrow w = \int F\,dx$$

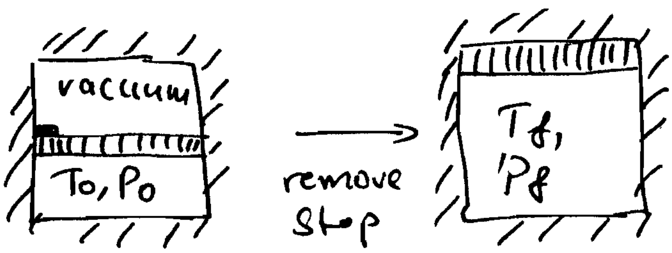
F, W depend on path

F, W not functions of x

$đw$ is an "inexact" differential

Example

Insulated tank w/ piston of mass m



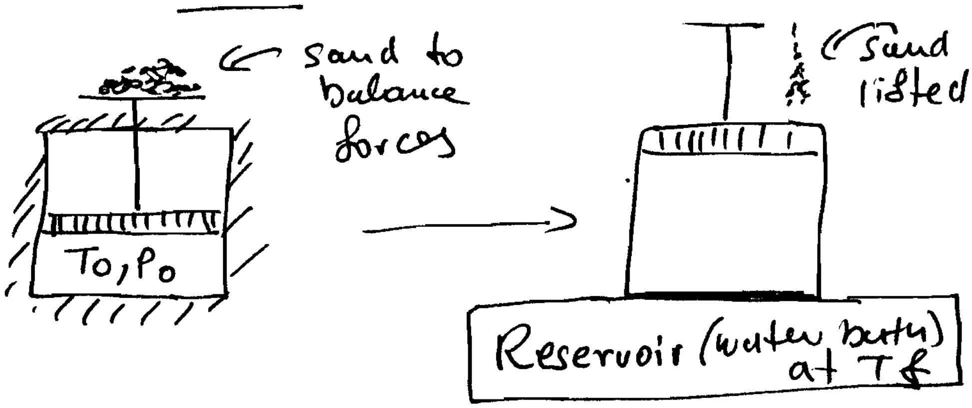
piston moves up, hits top wall + stops

Work is done by the gas on the piston

Net work to raise piston:

$W = -mg\Delta h$ [negative from point of view of system = gas]

The same final state can be reached as follows:



gas is at same V_f, T_f $\Rightarrow$ same P_f

More work was produced in the second case – where did it come from? → "Thermal" interactions

First Law (Energy Conservation)

The total energy E of a closed system is conserved:

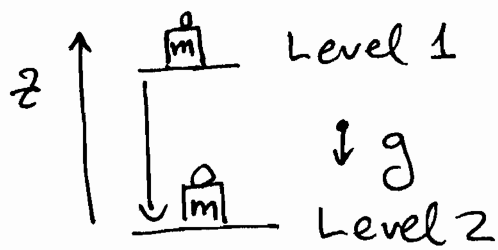
$\boxed{\Delta E = Q + W}$ Defines heat Q $\left\{\begin{array}{l}\text{adiabatic changes} \\ \text{used to measure } \Delta E\end{array}\right\}$

Signs: work, heat are positive when input to a system

Energy E: Potential, kinetic and internal (U)

(*) Potential energy: due to position of system in a force field

- e.g. gravitational



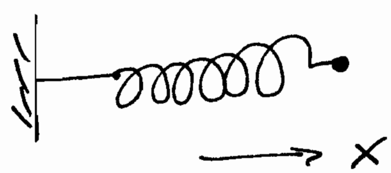
$$F = -mg$$

$$W = -\int_{z_1}^{z_2} mg\,dz = -mg(z_2 - z_1)$$

(if $z_2 < z_1$, $W > 0$, done on system)

- a linear spring

$F = -k(x - x_0)$ [by spring on environment]



$F_{ext} = -F$ [by env. on spring]

$$W = \int_{x_1}^{x_2} F_{ext}\,dx = \int_{x_1}^{x_2} k(x - x_0)\,dx =$$

[This could also be considered internal energy of spring]

$$E_{pot} = \frac{k}{2}(\Delta x_2^2 - \Delta x_1^2) \quad (\text{on spring})$$

(*) Kinetic energy: due to macroscopic motion:

$$E_{kin}(v) = \int F\,dx = \int m\frac{dv}{dt}v\,dt = \int_0^v mv\,dv = \frac{1}{2}mv^2$$

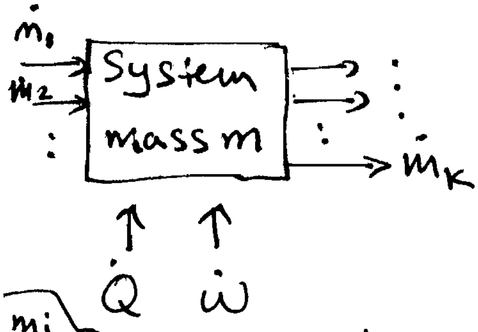
(*) Internal energy U: due to molecular motions + interactions

Many (but not all) systems of interest have small kinetic + potential energy changes relative to internal energy changes, so $E \simeq U$, $\boxed{\Delta U = Q + W}$

Differential form, $\boxed{dU = \delta Q + \delta W}$

Internal energy U is a function of thermodynamic state (unique $f(N, V, T)$ for 1-component system)

Open Systems



Signs: + for input
− for output

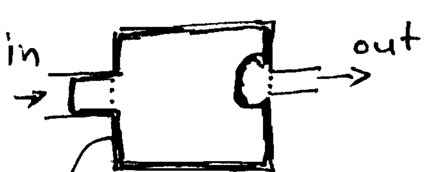
$$\frac{d}{dt}\left[U + m\left(\frac{v^2}{2} + gz\right)\right] = \left\{\begin{array}{c}\text{Rate of}\\ \text{net energy}\\ \text{input}\end{array}\right\}$$

$$= \dot{Q} + \dot{W} + \sum_{i=1}^{k} \dot{m}_i\left(\underline{U}_i + \frac{v_i^2}{2} + gz_i + P\underline{V}_i\right) \quad (i)$$

specific energy on mass basis, [J/kg]

why?

Need for $P\underline{V}_i$ term



equivalent Closed system

$$\dot{W}_{PV} = \frac{F\cdot\delta x}{\delta t} = \frac{P\cdot \underbrace{A\cdot\delta x}_{\delta V}}{\delta t} = P\underline{V}\cdot\dot{m}$$

Define enthalpy $\boxed{H = U + PV}$, substitute in (i).

$$\frac{d}{dt}\left[U + m\left(\frac{v^2}{2} + gz\right)\right] = \dot{Q} + \dot{W} + \sum_{i=1}^{k} \dot{m}_i\left(\underline{H}_i + \frac{v_i^2}{2} + gz_i\right)$$

In many (but <u>not</u> all) cases of interest, changes in potential + kinetic energy of the system and input/output streams can be neglected.

Then:

FIRST LAW DIFF. FORM

$$\frac{d\underline{U}}{dt} = \dot{Q} + \dot{W} + \underbrace{\sum_{\text{entering streams}} \underline{H}_{in} \dot{N}_{in} - \sum_{\text{leaving streams}} \underline{H}_{out} \dot{N}_{out}}_{\text{split entering/leaving, } \dot{N}_i > 0 \text{ in this form}}$$

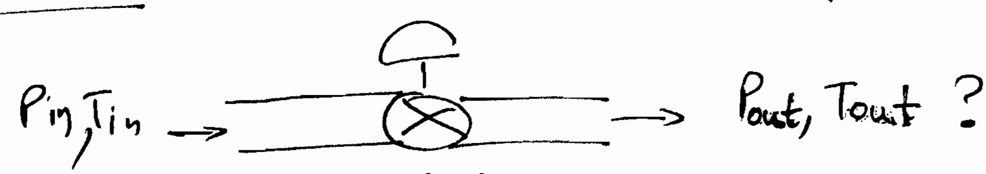
this refers to the system

If properties of entering + leaving streams are constant over time, <u>integrate</u>:

$$\underbrace{\Delta \underline{U}}_{\phi \text{ at steady-state}} = Q + W + \sum_{\text{entering}} \underline{H}_{in} N_{in} - \sum_{\text{leaving}} \underline{H}_{out} N_{out}$$

ϕ at <u>steady-state</u>

<u>Example</u> Joule-Thompson expansion



adiabatic "throttling" valve

$$\underbrace{\phi}_{\text{Steady-state}} = \underbrace{\cancel{Q}}_{\phi,\ \text{adiabatic}} + \underbrace{\cancel{W}}_{\phi,\ \text{no work}} + \underline{H}_{in} N_{in} - \underline{H}_{out} N_{out} \Rightarrow \boxed{\underline{H}_{in} = \underline{H}_{out}}$$