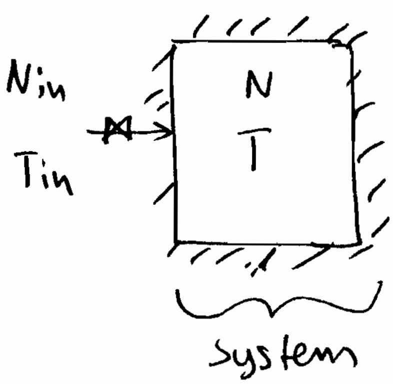
Problem Solving Strategy

1. Draw a schematic diagram, label key quantities
2. Define the thermodynamic system(s) of interest
3. State assumptions
4. Write down first Law balance
5. Apply equation-of-state (ideal gas) equation or obtain properties from NIST WebBook
6. Solve using symbols as far as possible

Example 2.5 - filling of gas cylinder



Insulated system, N_i, T_i

Pump in gas at T_{in} until N_f moles in tank

$T_f = ?$

* assume ideal gases, contents of cylinder at equilibrium as it is being filled

* $Q = 0$ (insulated) $\quad W = 0$

$$dU = \underline{H}_{in} \, dN_{in}$$

[$U = N\underline{U}$, both N and $\underline{U}$ are changing!]

$$d(N\underline{U}) = N d\underline{U} + \underline{U} dN = \underline{H}_{in} \, dN$$

$dN_{in} = dN$

For balances involving ideal gases, common (easiest) reference state to use is $\underline{u}=0$ at $T=0$ $\Rightarrow$ $\underline{H}=0$ @ $T=0$. Then

$$\underline{u} = C_v T \quad (\text{const. } C_v)$$
$$\underline{H} = C_p T \quad (\text{const. } C_p)$$

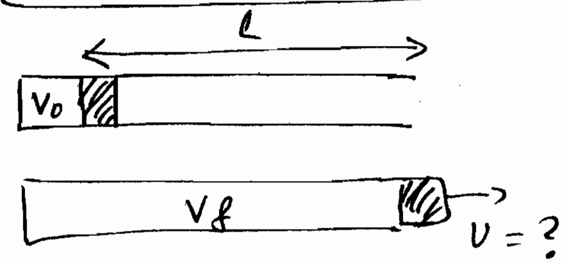
Apply here: $N C_v dT + C_v T dN = C_p T_{in} dN \Rightarrow$

$$\Rightarrow N C_v dT = (C_p T_{in} - C_v T) dN \Rightarrow$$

$$\int_{T_i}^{T_f} \frac{dT}{\gamma T_{in} - T} = \int_{N_i}^{N_f} \frac{dN}{N} \Rightarrow -\ln \frac{\gamma T_{in} - T_f}{\gamma T_{in} - T_i} = \ln \frac{N_f}{N_i} \Rightarrow$$

$$\Rightarrow \frac{\gamma T_{in} - T_f}{\gamma T_{in} - T_i} = \frac{N_i}{N_f} \Rightarrow T_f = \gamma T_{in} - \frac{N_i}{N_f}(\gamma T_{in} - T_i)$$

Example 2.8 - Air rifle



$V_o = 10 \text{ cm}^3$

$P_o = 3$ bar (absolute)

$\theta_o = 25\,^\circ C \Rightarrow T_o = 298\,K$

$m = 2g$ $\quad A = 0.3 \text{ cm}^2$

$\ell = 50$ cm

$C_v^{air} = 20.8 \text{ J/(mol K)} = \frac{5R}{2}$

$c_{v,proj} = 0.126 \text{ J/(g K)}$

$P_{atm} = 1$ bar

$V_f = V_o + A \cdot \ell = 25 \text{ cm}^3$

Rapid expansion $\rightarrow$ adiabatic, Eq. (2.30)

$$\left(\frac{T_f}{T_o}\right) = \left(\frac{V_f}{V_o}\right)^{-\frac{R}{C_v}} \Rightarrow T_f = 298\left(\frac{25}{10}\right)^{-\frac{2}{5}} K = 207\,K$$

Work: $\Delta U = \cancel{Q} + W \Rightarrow \Delta U = W \Rightarrow$

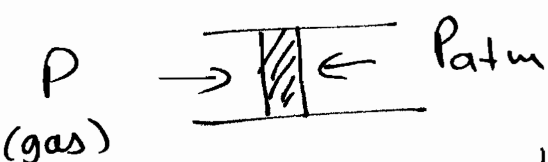
$$W = N c_v \Delta T = \frac{P_0 V_0}{R T_0} c_v \Delta T =$$

$$= \frac{3 \cdot 10^5 \, Pa \cdot 10 \cdot 10^{-6} \, m^3 \cdot 5/2}{298 \; K} \cdot (207 - 298) K =$$

$$= -2.30 \, (Pa \cdot m^3) = -2.30 \; J$$ (produced by gas)

[PV has units of work]

But... not all of this work goes into kinetic energy of the projectile



Net force on mass $(P - P_{atm}) \cdot A$

new system: projectile + atmosphere:

Work to "push back" atmosphere $-P_{atm} \Delta V$

(new W = − old W; new ΔV = − old ΔV)

$$W = \frac{1}{2} m v^2 - P_{atm} \Delta V \Rightarrow$$

$$v = \sqrt{\frac{2 (W + P_{atm} \Delta V)}{m}} = \sqrt{\frac{2 \cdot (2.30 \, J - 10^5 \, Pa \cdot 15 \cdot 10^{-6} \, m^3)}{0.002 \; kg}}$$

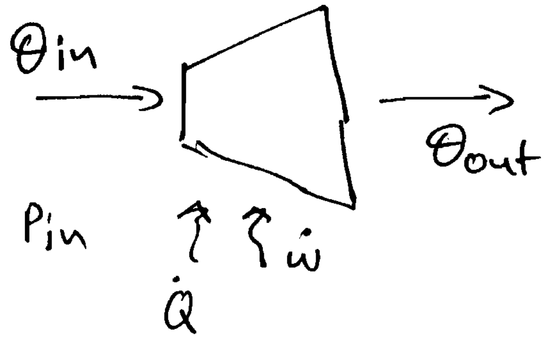
$$\Rightarrow v = 28.3 \; m/s$$

For projectile hitting a wall, assume all kinetic energy goes to heat it, $\Delta U = m c_v \Delta T = \frac{1}{2} m v^2 \Rightarrow$

$$\Delta T = 3.2 \; K$$

TOPS 35500

Example 2.9 Turbine Power



$\dot{Q} = -3\ \text{kW}$

$\theta_{in} = 200\ ^\circ C$

$P_{in} = 3\ \text{bar}$

$\dot{N} = 0.12\ \text{kg/s}$

$\theta_{out} = 140^\circ C$, saturated vapor

Steady-state:

$$\frac{dU}{dt} = 0 = \dot{Q} + \dot{W} + \dot{N}\left(\underline{H}_{in} - \underline{H}_{out}\right) \Rightarrow$$

$$\dot{W} = -\dot{Q} + \dot{N}\left(\underline{H}_{out} - \underline{H}_{in}\right)$$

$$\left.\begin{array}{l} \underline{H}_{out}\ (\text{satd.},\ 140^\circ C) = 2733.4\ \text{J/kg} \\ \underline{H}_{in}\ (3\ \text{bar},\ 200^\circ C) = 2865.9\ \text{J/kg} \end{array}\right\}$$ from WebBook

$$\dot{W} = +3\ \text{kW} + 0.12\ \frac{\text{kg}}{\text{s}} \cdot (2733.4 - 2865.9)\ \frac{\text{kJ}}{\text{kg}} \Rightarrow$$

$$\boxed{\dot{W} = -12.9\ \text{kW}}$$