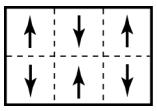
Problem Set # 3

Due @ 11:59 pm Tuesday 3/12 by electronic submission on BlackBoard

1. A simple model for magnetic materials is the "Ising model," consisting of spins with nearest-neighbor interactions on a lattice. For the 2×3 square lattice shown on the right, calculate (a) the total number of microstates possible and (b) the entropy as a function of the energy for all possible energies. Each interaction between neighbors to the left, right, above, or below results in energy of $-k_B T_0$ if the spins are parallel, and $+k_B T_0$ if the spins are antiparallel. For example, the microstate above has energy of $+7k_B T_0$. The boundary walls do not interact with the spins.



2. A fluid has the following canonical partition function, where a and b are constants:

$$\mathcal{Q}(N,V,T)=\frac{(V-Nb)^N (k_B T)^{5N/2}}{N!}\exp\left(\frac{aN^2}{k_B TV}\right)$$

Obtain the energy, pressure and heat capacity at constant volume of this fluid as a function of molar volume and temperature, $\underline{U}=\underline{U}(\underline{V},T)$, $P=P(\underline{V},T)$, and $C_V=C_V(\underline{V},T)$.

3. The semigrand canonical ensemble for a two-component system is defined as the one at constant μ_1, N_2, V and T. Obtain expressions for the partition function and the probabilities of microstates in this ensemble. Which thermodynamic function (or combination of thermodynamic functions) corresponds to the partition function of this ensemble?

4. Consider a system of N spheres similar to that shown in Figure 1 (page 2) of the notes, except with only 2 energy levels, 0 and $+k_B T_0$.

   (a) Obtain a general expression for the number of microstates Ω and the temperature T as a function of the total energy U. [*Hint* : to obtain T, use Stirling's approximation, $\ln(N!)\approx N\ln N - N$, valid for large N].

   (b) What is the temperature for $U = Nk_B T_0/2$? What is the sign of temperature for energies greater than $Nk_B T_0/2$? Are these values physically realistic?

5. Consider the system of 5-bead square-lattice chains discussed in Example 1 (p. 5) of the online notes (http://paros.princeton.edu/cbe422/StatMech.pdf). Obtain the average end-to-end distance (the distance between first and last bead) in units of the lattice spacing, at the two limits (a) $T\to 0$ (b) $T\to\infty$.

Please submit your results as a single PDF file. We will assign 20% of the grade to the "aesthetics" of your solution: how easy it is to follow, quality and proper labeling of graphs, etc.