

## Electron Spin

Electrons have an intrinsic angular momentum  $m_s$ , with spin quantum number  $S = +1/2$  (" $\alpha$ " electrons) or  $S = -1/2$  (" $\beta$ " electrons)

## Pauli Exclusion Principle

Electrons are Fermions (like protons & neutrons) and can only occupy the same orbital if their spins are opposite. Thus, each orbital ( $1s, p_x, \dots$ ) can accommodate up to two electrons (a pair).

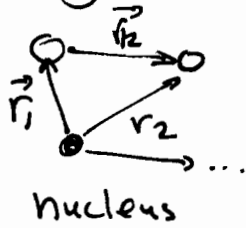
By contrast, photons ( $S=0$ ) or deuterons are Bosons and can occupy the same state, leading to Bose-Einstein condensation (Nobel 2001)<sup>phys.</sup>

## Spin Multiplicity

When there are unpaired electrons (e.g. in free radicals such as  $\text{O}\cdot$ ), the quantity  $2S + 1$  ( $S$  is the total angular spin momentum) determines filling of multiple orbitals of the same energy -

E.g. if  $S=0$  (no unpaired electrons)  $2S+1=1$  (singlet)  
 $S=1/2$  (1 unpaired)  $2S+1=2$  (doublet)  
 $S=1$  (2 unpaired)  $2S+1=3$  (triplet)

## Many-electron atoms



nucleus  
charge  $Ze$

$n$   
electrons  
mass  
 $m_e$

assume stationary nucleus  
(mass  $\gg m_e$ )

Hamiltonian for Schrödinger's equation:

$$\hat{H} = \sum_{i=1}^n \left( -\frac{\hbar^2}{2m_e} \nabla_i^2 - \frac{Ze^2}{4\pi\epsilon_0 r_i} \right) + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{e^2}{4\pi\epsilon_0 r_{ij}}$$

one-electron operator  $\hat{h}^{(i)}(\vec{r}_i) = -\frac{\hbar^2}{2m_e} \nabla_i^2 - \frac{Ze^2}{4\pi\epsilon_0 r_i}$

two-electron operator  $\hat{g}(\vec{r}_i, \vec{r}_j) = \frac{e^2}{4\pi\epsilon_0 r_{ij}}$

Total operator  $\hat{H} = \sum_{i=1}^n \hat{h}^{(i)}(\vec{r}_i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \hat{g}(\vec{r}_i, \vec{r}_j)$

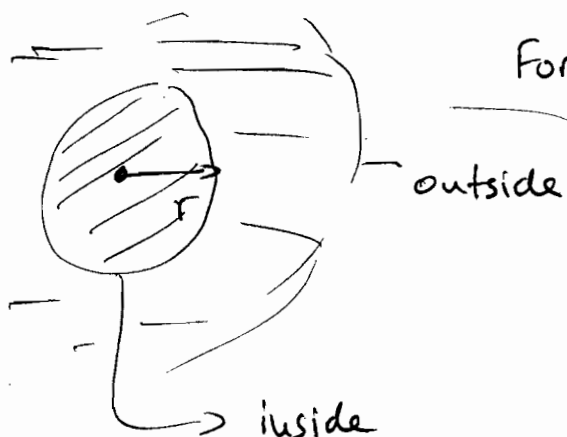
To solve:  $\hat{H}\psi = E\psi$

Need to make approximations because of the  $\hat{g}(\vec{r}_i, \vec{r}_j)$  terms. In the "orbital" approximation each electron occupies its "own" orbital

$$\psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n) \approx \psi(\vec{r}_1) \cdot \psi(\vec{r}_2) \dots \psi(\vec{r}_n)$$

where the orbitals are similar to those of the H atom, modified by the presence of all other electrons

Effective charge of nucleus:  $z_{\text{eff}} \cdot e < Z \cdot e$



For electron at mean distance  $r$ , the effect of electrons at distances less than  $r$  is to reduce the effective charge of the nucleus.

This approach is very approximate and only works for atoms.

### Aufbau Principle (building-up)

Orbitals fill up in succession:

	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s
# of orbitals	1	1	3	1	3	1	5	3	1	5	3	1

Each orbital can be occupied by up to 2 electrons (of opposite spin  $\downarrow\uparrow$ )

Hund's rule: An atom in its ground state adopts a configuration with the greatest number of unpaired electrons

Example - Na atom OMF calculation  
 $\text{Na}^+$  emission  
 energy  $\leftrightarrow$  wavelength  $E = \frac{hc}{\lambda}$