Work Interactions

A work interaction between two systems occurs when their boundary moves under the action of a force.

\[ \Delta W = F \Delta x \Rightarrow W = \int F \, dx \]

\( F, W \) depend on path,
\( F, W \) not functions of \( x \)
\( \Delta W \) is an "inexact" differential

Example

Insulated tank with piston of mass \( m \)

\begin{align*}
\text{vacuum} & \quad \text{remove stop} \\
T_0, P_0 & \quad \text{Piston moves up, hits top wall + stops}
\end{align*}

Work is done by the gas on the piston.
Net work to raise piston:

\[ W = -mg \Delta h \quad \text{[negative from point of view of system = gas]} \]

The same final state can be reached as follows.

Sand to balance forces

\begin{align*}
\text{Sand lifted} & \quad \text{gas is at same } V_f, T_f \\
& \Rightarrow \text{same } P_f
\end{align*}

Reservoir (water bath) at \( T_f \)

More work was produced in the second case - where did it come from? \( \Rightarrow \) "Thermal" interaction
First Law (Energy Conservation)

The total energy $E$ of a closed system is conserved:

\[ \Delta E = Q + W \]

Defines heat $Q$ {adiabatic changes, used to measure $\Delta E$}

Signs: work, heat are positive when input to a system

Energy $E$: Potential, kinetic, and internal ($U$)

1. Potential energy: due to position of system in a field
   - e.g. gravitational

   \[
   F = -mg \\
   \begin{array}{c}
   m \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   z \uparrow \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   \hline
   z \downarrow \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   g \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   \hline
   m \quad \text{Level 1} \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   \hline
   \text{Level 2} \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   \hline
   z_2 \quad \text{Level 2} \\
   \hline
   \hline
   \end{array}
   \quad \begin{array}{c}
   \hline
   z_1 \quad \text{Level 1} \\
   \hline
   \hline
   \end{array}
   \]

   \[
   W = -\int mgdz = -mg(z_2 - z_1) \\
   (\text{if } z_2 < z_1, \text{ work done on system})
   \]

2. A linear spring

   \[
   F = -K(x-x_0) \quad \text{[by spring on environment]} \\
   \]

   \[
   F_{ext} = -f \quad \int [\text{by env. on spring}] \\
   \]

   \[
   W = \int f_{ext} dx = \int K(x-x_0)dx = \\
   \]

   \[
   E_{pot} = \frac{K}{2} (\Delta x_2^2 - \Delta x_1^2) \quad \text{[on spring]} \\
   \]

3. Kinetic energy: due to macroscopic motion:

   \[
   E_{kin}(v) = \int Fdx = \int m \frac{dv}{dt}udt = \int mvudv = \frac{1}{2} mv^2
   \]

   \[
   \]
Internal energy \( U \): due to molecular motions + interactions

Many (but not all) systems of interest have small kinetic + potential energy changes relative to internal energy changes, so \( E \approx U \), \( \Delta U = Q + w \)

Differential form, \( dU = dQ + dw \)

Internal energy \( U \) is a function of thermodynamic state (unique \( S(N, V, T) \) for 1-component system)

Open Systems

\[
\frac{d}{dt} \left[ U + m \left( \frac{v_i^2}{2} + g_z \right) \right] = \left\{ \begin{array}{l} \text{Rate of net energy input} \\ \text{specific energy on mass basis, [J/kg]} \end{array} \right\}
\]

\[= Q + \dot{w} + \sum_{i=1}^{n} \frac{k}{m_i} \left( U_i + \frac{v_i^2}{2} + g_z i + P_v i \right) \]

Need for \( PV \) term in equivalent closed system

Define enthalpy \( H = U + PV \), substitute in (i).

\[
\frac{d}{dt} \left[ U + m \left( \frac{v_i^2}{2} + g_z \right) \right] = \dot{Q} + \dot{w} + \sum_{i=1}^{k} \frac{k}{m_i} \left( \frac{H_i + v_i^2}{2} + g_z i \right)
\]
In many (but not all) cases of interest, changes in potential and kinetic energy of the system and input/output streams can be neglected.

Then:

\[
\frac{\Delta u}{\Delta t} = q + w + \sum \frac{H_{in} N_{in}}{N_{in}} - \sum \frac{H_{out} N_{out}}{N_{out}}
\]

First Law
Diff. Form

\[\text{this refers to the system}\]

\[\text{Split entering/leaving}\]
\[\text{\(N_i > 0\) in this form}\]

If properties of entering and leaving streams are constant over time, integrate:

\[\Delta u = q + w + \sum \frac{H_{in} N_{in}}{N_{in}} - \sum \frac{H_{out} N_{out}}{N_{out}}\]

\[\emptyset\text{ at steady-state}\]

Example: Joule-Thompson expansion

\[P_{in}, T_{in} \rightarrow \text{adiabatic} \rightarrow \text{"throttling" valve} \rightarrow P_{out}, T_{out}\]

\[\emptyset = \emptyset + \emptyset + \frac{H_{in} N_{in} - H_{out} N_{out}}{w}\]

\[\emptyset = \emptyset, \text{adiabatic}\]

\[\emptyset, \text{no work}\]

\[H_{in} = H_{out}\]