Problem Solving Strategy

1. Draw a schematic diagram, label key quantities
2. Define the thermodynamic system(s) of interest
3. State assumptions
4. Write down first law balance
5. Apply equation-of-state (ideal gas) equation, or obtain properties from NIST Webbook
6. Solve using symbols as far as possible

Example 2.5 - Filling a gas cylinder

\[
N_{\text{in}}, T_{\text{in}}\quad \text{Insulated system, } N_i, T_i
\]

Pump in gas at \( T_{\text{in}} \) until \( N_f \) moles in tank

\( T_f = ? \)

* Assume ideal gases, contents of cylinder at equilibrium as it is being filled

* \( Q = 0 \) (insulated) \( W = 0 \)

\[
dU = H_{\text{in}} dN_{\text{in}} \quad \left[ U = N U_i \text{, both } N \text{ and } U \text{ are changing!} \right]
\]

\[
d(NU) = N \text{d}U + U \text{d}N = H_{\text{in}} \text{d}N
\]

\[
\uparrow \quad \text{d}N_{\text{in}} = \text{d}N
\]
For balances involving ideal gases, common (easiest) reference state to use is \( u = 0 \) at \( T = 0 \) ⇒ \( h = 0 \) @ \( T = 0 \). Then \( u = C_v T \) (const. \( C_v \)) 
\( H = C_p T \) (const. \( C_p \))

**Apply here:** \( N C_v dT + C_v T dN = C_p T_{in} dN \) ⇒

\[ \Rightarrow N C_v dT = (C_p T_{in} - C_v T) dN \] ⇒

\[ \int_{T_i}^{T_f} \frac{dT}{T_{in}-T} = \int \frac{kN}{N} = -\ln \frac{T_{in}-T_f}{T_{in}-T_i} = \ln \frac{N_i}{N_f} \] ⇒

\[ \frac{T_{in}-T_f}{T_{in}-T_i} = \frac{N_i}{N_f} \] ⇒

\[ T_f = T_{in} - \frac{N_i}{N_f} (T_{in}-T_i) \]

**Example 2.8 - Air rifle**

\[ V_0 = 10 \text{ cm}^3 \]
\[ P_0 = 3 \text{ bar} \text{ (absolute)} \]
\[ \theta_0 = 25^\circ C \Rightarrow C_{b0} = 298 K \]
\[ m = 2 g \]
\[ A = 0.3 \text{ cm}^2 \]
\[ L = 50 \text{ cm} \]
\[ P_{atm} = 1 \text{ bar} \]

\[ V_f = V_0 + A \cdot L = 25 \text{ cm}^3 \]

**Rapid expansion \( \rightarrow \) adiabatic**, Eq. (2.30)

\[ \left( \frac{T_f}{T_0} \right) = \left( \frac{V_f}{V_0} \right)^{-\frac{R}{C_v}} \Rightarrow T_f = 298 \left( \frac{25}{10} \right)^{-\frac{2}{5}} \approx 207 \text{ K} \]
Work: \[ \Delta U = Q + W \Rightarrow \Delta U = W \Rightarrow \]

\[ w = Nc_v \Delta T = \frac{p_0 v_0}{R} c_v \Delta T = \frac{3 \times 10^5 \text{Pa} \cdot 10 \cdot 10^{-6} \text{m}^3 \cdot 5/2}{298 \text{ K}} \cdot (207 - 298) \text{ K} = -2300 \text{ (Pa} \cdot \text{m}^3) = -2300 \text{ J} \quad \text{(produced by gas)} \]

\[ PV \text{ has units of work} \]

But... not all of this work goes into kinetic energy of the projectile

\[ P \rightarrow \text{Net force on mass} \]

\[ \text{(gas)} \quad \text{(P - Patm) A} \]

\[ \text{new system: projectile + atmosphere: work to "push back" atmosphere} \ - \ \text{Patm} \Delta V \]

\[ W = \frac{1}{2} m v^2 - \text{Patm} \Delta V \Rightarrow \]

\[ v = \sqrt{\frac{2 (W + \text{Patm} \Delta V)}{m}} = \sqrt{\frac{2 \cdot 2300 \text{ J} \cdot 10^5 \text{Pa} \cdot 15 \text{ m}^2}{0.002 \text{ kg}}} \]

\[ \Rightarrow v = 28.3 \text{ m/s} \]

For projectile hitting a wall, assume all kinetic energy goes to heat if, \[ \Delta U = mc_v \Delta T = \frac{1}{2} \text{ m} v^2 \Rightarrow \]

\[ \Delta T = 3.2 \text{ K} \]
Example 2.9 Turbine Power

\[ \dot{Q} = -3 \text{ kW} \]

\[ \Theta_{\text{in}} = 200 \degree \text{C} \]
\[ P_{\text{in}} = 3 \text{ bar} \]
\[ \dot{N} = 0.12 \text{ kg/s} \]
\[ \Theta_{\text{out}} = 140 \degree \text{C}, \text{ saturated vapor} \]

Steady-state:

\[ \frac{du}{dt} = 0 = \dot{Q} + \dot{W} + \dot{N}(\dot{h}_{\text{in}} - \dot{h}_{\text{out}}) \Rightarrow \]

\[ \dot{W} = -\dot{Q} + \dot{N}(\dot{h}_{\text{out}} - \dot{h}_{\text{in}}) \]

\[ \dot{h}_{\text{out}}\left(\text{saturated, 140°C}\right) = 2733.40 \text{ kJ/kg} \]
\[ \dot{h}_{\text{in}}\left(3 \text{ bar, 200°C}\right) = 2865.9 \text{ kJ/kg} \]

\[ \dot{W} = +3 \text{ kW} + 0.12 \cdot \frac{\text{kJ}}{\text{s}} \cdot \left(2733.4 - 2865.9\right) = \]

\[ \boxed{\dot{W} = -12.9 \text{ kW}} \]