All reversible engines operating between given $\Theta_h$, $\Theta_c$ with same $\Delta Qc$ have the same $\Delta W_1, \Delta Q_H$.

\[ \frac{|Q_H^{\text{rev}}|}{|Q_C^{\text{rev}}|} = f(\Theta_h, \Theta_c) \]  

\text{A universal function}

Moreover, from a "cascade" of reversible engines, for any $\Theta_m$,

\[ |Q_c| = |Q_c'| \Rightarrow \frac{|Q_H|}{f(\Theta_m, \Theta_c)} = \frac{|Q_H'|}{f(\Theta_m, \Theta_c)} \]

\[ \frac{|Q_H|}{f(\Theta_h, \Theta_m)} f(\Theta_m, \Theta_c) = \frac{|Q_H'|}{f(\Theta_h, \Theta_m)} f(\Theta_m, \Theta_c) \]

\[ \Rightarrow f(\Theta_h, \Theta_c) = f(\Theta_h, \Theta_m) f(\Theta_m, \Theta_c) \]

Two possibilities:

1. \[ f(\Theta_h, \Theta_c) = \frac{g(\Theta_h)}{g(\Theta_c)} \quad \text{or} \quad \frac{g(\Theta_c)}{g(\Theta_H)} \]

2. \[ g(\Theta) = T \text{ defines the thermodynamic temperature} \]

\[ T, \text{ to be shown in Sec.1 to be equivalent to the ideal-gas temperature} \quad T = \frac{\nu}{k} \]

\[ g(\Theta) = \frac{1}{kT} \equiv \beta \text{ is used in statistical mechanics} \]
Now, the ratio at the top of p. 5 can be written as:

\[ \frac{|Q_H|}{|Q_C|} = \frac{T_H}{T_C} \Rightarrow \frac{Q_H}{T_C} = \frac{T_H}{T_C} \]

\[ \Rightarrow \frac{Q_H}{T_H} + \frac{Q_C}{T_C} \]

\[ \text{Efficiency of reversible (Carnot) engine:} \quad \eta_{\text{rev}} = \frac{-W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{T_H - T_C}{T_H} \]

\[ 1 \omega_1 = \eta_{\text{rev}} |Q_H| \]

For refrigeration cycle, one uses work to remove heat from the interior of a device or building;

coefficient-of-performance \( J \) (Zeta):

\[ J = \frac{|Q_C|}{1 \omega_1} = \frac{Q_C}{-Q_C - Q_H} = \frac{Q_C}{-Q_C + \frac{T_H}{T_C} Q_C} \]

\[ \Rightarrow J = \frac{T_C}{T_H - T_C} \quad \text{can be} \quad J > 1 \]

Another possibility is operation as a "heat pump" to warm a building by transferring heat from the cold outside air. In that case,

\[ J = \frac{-Q_H}{1 \omega} = \frac{-Q_H}{-Q_C - Q_H} = \frac{Q_H}{Q_H - \frac{T_C}{T_H} Q_H} = \frac{T_H}{T_H - T_C} \quad \text{(also be } J > 1) \]
Example 3.2
Maximum $J$ for air conditioner $\Theta_{in} = 68^\circ F$ $\Theta_{out} = 104^\circ F$

$$J_{rev} = \frac{Q_c}{W} = \frac{T_c}{T_h - T_c} = \frac{293}{(313 - 293)K} = 14.6$$

**Entropy**

We have just showed that for reversible processes, $$\frac{Q_{rev}}{T_h} + \frac{Q_{rev}}{T_c} = 0 \text{ for complete engine "cycle"}$$

This suggests the possibility of defining a new thermodynamic quantity (state function) related to the reversible heat exchanged:

$$dS = \frac{\delta Q_{rev}}{T} \quad \Delta S = S_B - S_A = \int_A^B \frac{\delta Q_{rev}}{T}$$

**Key Property of $S$**:

- $\Delta S = 0$ for reversible processes
- $\Delta S > 0$ for spontaneous (irreversible) ones in isolated systems

**Proof**

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**irreversible path** $A \rightarrow D$

- $B-C$ isothermal
- $A-B-C-D$ adiabatic reversible process
- $D \rightarrow A$
CBE 246 Second Law

\[ U_A - U_D = Q_{\text{rev}} + w_{\text{rev}}^{D\rightarrow A} = T(S_A - S_D) + w_{\text{rev}}^{D\rightarrow A} \]  \( \tag{1} \)

For irreversible process \( A \rightarrow D \) (closed, isolated system)

\[ U_0 - U_A = w_{\text{irr}}^{A\rightarrow D} \]  \( \tag{2} \)

\( 1 + 2 \Rightarrow T(S_A - S_D) + w_{\text{rev}}^{D\rightarrow A} + w_{\text{irr}}^{A\rightarrow D} = 0 \)

If \( w_{\text{irr}}^{A\rightarrow D} + w_{\text{rev}}^{D\rightarrow A} < 0 \) (work output)

\[ T(S_A - S_D) = Q_{\text{rev}}^{D\rightarrow A} > 0 \] (heat absorbed)

\[ \Rightarrow \text{violates Kelvin-Planck postulate!} \]

Therefore, \( w_{\text{irr}}^{A\rightarrow D} + w_{\text{rev}}^{D\rightarrow A} > 0 \)

(\( = 0 \) would be reversible both ways)

\[ S_A - S_D < 0 \Rightarrow [S_D > S_A] \] For irreversible process \( A \rightarrow D \)

Final state had higher entropy!

Also, \( w_{\text{irr}}^{A\rightarrow D} > w_{\text{rev}}^{A\rightarrow D} \)

If work is produced \( +w_{\text{rev}}^{A\rightarrow D} < 0 \)

Irreversible work is less in absolute magnitude (more positive)

If work is needed, irreversible work required is more. \( \Rightarrow \) Reversible processes are "the best!"